exist a very few cases when the change in components is small where different calculations can lead to uncertainty in the direction of the changes in the scaled components. It should be emphasized that such failings will be rare, and in these cases the problem generally lies in not knowing the precise geometries rather than with the component analysis. For most cases, this method provides a convenient, reliable, and insightful way to describe conformational changes.

Conclusion

Conformational behavior can generally be interpreted in terms of a component analysis. For hydrocarbons or systems containing only a single heteroatom when atoms or groups are moved into closer proximity, V_{ee} and V_{nn} increase and V_{ne} decreases. The change in the total energy ΔE is small, and its direction depends upon the relative magnitudes of the various terms. If, for the scaled components, the sum $\Delta V_{ee} + \Delta V_{nn}$ is larger, a repulsive interaction exists between the groups. When ΔV_{ne} is larger, there exists an attractive steric effect, and the system is more stable with the bulky groups closer.

Even for those cases which are dominated by the repulsive terms, a component analysis provides greater insight into the magnitude of the energy changes and the causes of conformational preferences.

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An Intramolecular Hydrogen Bond. Ab Initio MO Calculations on the Enol Tautomer of Malondialdehyde

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Abstract: Ab initio MO-LCAO-SCF calculations have been performed on the enol tautomer of malondialdehyde, the parent compound for β-dicarbonyls. The molecule contains an intramolecular hydrogen bond, the potential of which is found to be of the double-well type with a barrier height of 48 kJ/mol. The results of the calculations are presented with reference to experimental data on β-dicarbonyl compounds.

The chemical phenomenon of hydrogen bonding has been studied extensively by quantum-mechanical ab initio calculations.[2] These investigations have focussed primarily on *inter*molecular hydrogen bonding. Rather few calculations have dealt with *intra*molecular hydrogen bonds.[3,4] One reason for this seems to be the comparatively large molecular framework required to make an intramolecular hydrogen bond possible. This has tended to make such calculations rather time-consuming. However, the use of new efficient computer programs has reduced these calculation times considerably.

It is generally assumed[5] that the relative stability of enol tautomers of β-dicarbonyl compounds derives partially from the formation of an intramolecular hydrogen bond. Some controversy reigns as to whether such a bond is strong enough to have the proton effectively centered in a single-minimum potential. The problem is exemplified by malondialdehyde, which is best represented either by an asymmetric C_s structure (Ia and Ib in Figure 1) or by a symmetric C_{2v} structure (II).

We have undertaken a series of ab initio MO calculations on the enol tautomer of malondialdehyde in order to gain information about the potential energy surface relevant to the proton transfer process, and in particular to distinguish between the forms I and II of Figure 1. Since the quadrupole coupling constant of a deuterium atom is quite sensitive to the bonding situation, it might be used in the future for an experimental resolution of the question of symmetry. We have therefore computed this quantity for a deuterium atom in the hydrogen bond, assuming various molecular geometries.

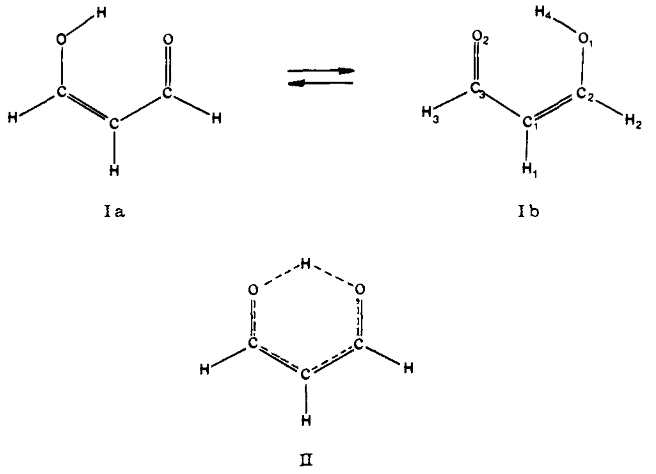

Figure 1. Possible structural representations of the tautomer of malondialdehyde. Form I has C_s symmetry with a double minimum hydrogen bond potential; form II has C_{2v} symmetry with a single minimum potential. The labeling of the atoms is shown in Ib.

Calculations

Basis Set. The ab initio calculations were performed with the computer program MOLECULE.[6] A Gaussian basis set consisting of seven s- and three p-type functions on C and O[7] and four s type on H[8] was contracted to 4s, 2p and 2s, respectively. In addition, a group of p functions with exponents 0.8 was used for the H atom in the hydrogen bond in order to introduce some additional flexibility to the basis set in the region of particular interest.[9] The basis set thus contained in all 99 uncontracted and 61 contracted basis functions.

Geometrical Optimization. Since our major aim was to determine the energy difference between the C_s and the C_{2v} form of malondialdehyde we considered it mandatory to perform a geometry optimization. Malondialdehyde has 21 internal degrees of freedom so that certain constraints on the geometry had to be applied in order to reduce the computational effort. This was accomplished in the following manner. (i) In accordance with all available X-ray and electron diffraction data on analogous β-dicarbonyl compounds[10-13] the molecule was considered planar. (ii) All C-H bond lengths were set equal to 1.10 Å. (iii) The ratios between angles $\angle O_1C_2H_2:\angle C_1C_2H_2$, $\angle O_2C_2H_3:\angle C_1C_3H_3$, and $\angle C_2C_1H_1:\angle C_3C_1H_1$ were kept constant (for the atom labeling see Figure 1) and equal to the corresponding values determined by electron diffraction[13] on the enol form of 2,4-pentanedione (acetylacetone, in the following acac). While the assumption i is supported by experiment the constraints ii and iii are more arbitrary.

For the present purpose, however, it is clear from standard values of vibrational force constants that quite large uncertainties in these angles and bond distances can be tolerated without affecting the energy comparison between the two forms of malondialdehyde. (An error of 7° in a CCH angle or 0.05 Å in a CH distance would produce an energy error of 4 kJ/mol (1 kJ = 0.239 kcal), assuming CH bending and stretching frequencies of 1100 and 2900 cm^{-1}, respectively.) The simplifying assumptions of i, ii, and iii reduce the number of independent internal parameters to nine for the C_s and five for the C_{2v} form.

It appeared reasonable to use one of the experimentally determined geometries of the acac enol[13] as a starting point for the optimization of the C_{2v} form II. There was, however, no natural choice of a starting geometry for C_s form I, a problem which was solved in the following manner.

(a) Exploratory CNDO/2 studies had given strong indications of a nonlinear hydrogen bond, contrary to the electron diffraction determinations.[12,13] The position of the atom H_4 in the C_{2v} form II was therefore optimized. A nonlinear hydrogen bond was obtained, with an energy gain of 60 kJ/mol.

(b) A limited variation in the geometry was performed in order to investigate the possibility of an energy minimum for the C_s form. The two parameters varied were the $C_3C_1C_2$ angle and a reaction coordinate k'. k' specifies the structural parameters according to the formula $p = k'p_{st} + (1 - k')p_{C_{2v}}$ where p is a bond length or bond angle, p_{st} is a corresponding value assuming localized single and double bonds,[14] and $p_{C_{2v}}$ is the experimental[13] value for acac with a modified H_4 position. In this way an approximate energy minimum was found for the C_s form with an energy gain of 39 kJ/mol.

(c) The structure so obtained was taken as a basis for a second optimization, now treating all the nine internal parameters as independent. The parameters were chosen as bond lengths and bond angles and the optimization was performed in the valence force field approximation, i.e., by varying one parameter at a time starting each time from the basic structure. Coupling between the chosen variables was neglected, and parabolic potential was assumed in the calculation of the optimal value for each parameter. The energy was calculated at points differing in 0.03 Å and 2° for bond lengths and bond angles, respectively, and it was ensured that the optimal value occurred within the interval investigated. It turned out that the structure obtained in step b was rather close to the optimum obtained in step c except for the C_2-O_1 bond which changed by 0.077 Å. This change accounted for nearly half of the total energy gain of 22 kJ/mol in this optimization cycle.

(d) The optimal energy of the C_{2v} symmetric form II was also calculated to investigate the barrier of topomerization. With the constraint of C_{2v} symmetry, there remained only five variables in the optimization of the geometry. This was performed using the methods described in step c above. The optimization yielded an energy gain of 13 kJ/mol.

Results. It was found that the C_s form I was more stable that the C_{2v} form II by 48 kJ/mol. The calculated optimal geometries for the two forms are shown in Figure 2. The results of an electron diffraction determination of the acac enol[13] are also provided for comparison.

The hydrogen bond potential was calculated as a function of a reaction coordinate giving the structural parameters as a linear interpolation of the calculated final values for the forms I and II. The resulting potential is shown in Figure 3.

Discussion

Accuracy of the Calculations. Several errors are inherent in the present calculations. The basis set used is of interme-

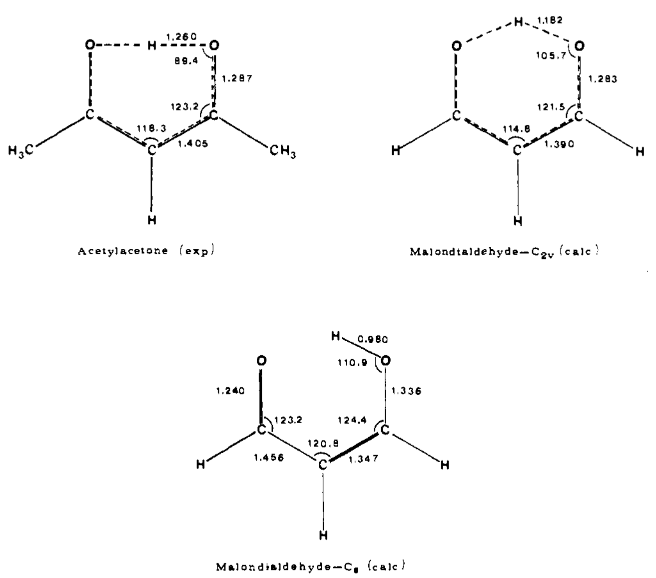

Figure 2. Calculated geometries of malondialdehyde. One of the experimentally determined structures of acac (from ref 12) is included for comparison.

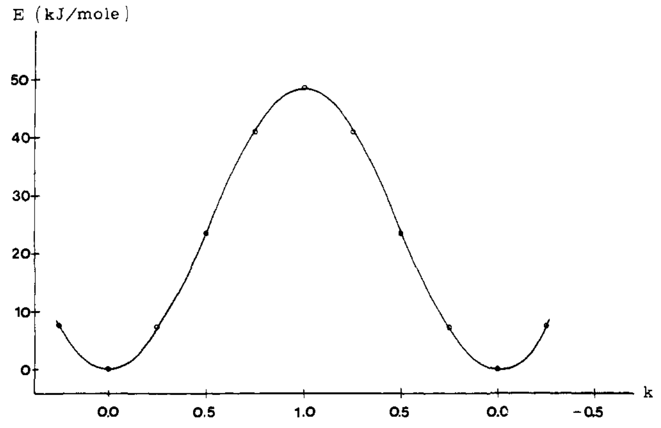

Figure 3. Calculated hydrogen bond potential. The reaction coordinate k determines the structure through a linear interpolation between the calculated optimal geometries so that $k = 0$ gives the optimal C_s and $k = 1$ the optimal C_{2v} structures.

diate size, and the possibility cannot be excluded that a larger basis set is needed to better describe, for example, the oxygen-oxygen interaction. We found that a calculation using a minimal basis set[15] gave no significant energy difference between the two forms I and II; this shows that the problem under consideration is sensitive to the choice of basis set.

A serious (and at the same time methodologically interesting) error is the one arising from correlation effects. It is well known that near degeneracy effects increase with increasing bond length. One can thus expect the correlation energy to be greater in the C_{2v} than in the C_s form because of the longer O-H distance. This would tend to lower the barrier for proton transfer, an effect which has indeed been found in the hydrogen bonds of the systems $H_5O_2^+$, $H_3O_2^-$, and FHF^-, where barriers of the order of 40 kJ/mol were lowered by 30% when correlation effects were included.[16,17] One might also anticipate that the inclusion of correlation effects would give calculated optimal geometries with more equal values for the two C-O and the two C-C bonds and a smaller $C_2C_1C_3$ angle in the C_s form.

A further error is introduced by the incomplete geometry optimization. The constraints i-iii placed on the structure should not introduce any serious errors in the energy comparison as argued above. It is, however, also clear that the optimization of the other parameters is not carried to convergence. For a reasonable energy surface it is expected that a further optimization cycle should give an energy gain that is considerably smaller than the one obtained in the previous cycle. The errors introduced by the incomplete optimization are thus most certainly small (of the order of a few kJ/mol) and should not affect the conclusions about the relative stability of the two forms.

Geometry. Malondialdehyde (MA) is unfortunately a rather reactive molecule and has so far only been studied in solution. No structure determination of MA has been reported and few reliable physical data are available. Other β-dicarbonyl compounds have been studied extensively,[5] however, and the present calculation is intended to contribute to the understanding of some of the general properties of these compounds. When structure permits it, the intramolecularly hydrogen-bonded enol form is usually the most stable tautomer for β-dicarbonyl compounds. Such a hydrogen bond is considered to be rather strong. A much debated question is, therefore, whether this bond is strong enough to have a single minimum hydrogen-bond potential, or is of the double-minimum type making the molecule skew symmetrical with distinct "alcohole" type and "carbonyl" type oxygens.

Experimental structure determinations made on enols of β-dicarbonyl compounds differ on the issue of molecular symmetry. Lowrey et al.[12] and Andreassen and Bauer[13] deduced the enol form of acac to possess C_{2v} symmetry on the basis of electron diffraction studies. An analogous result was obtained by Schaefer and Wheatley[11] in an X-ray investigation of crystalline 3,4-diaceto-2,5-hexanedione. Williams,[10] also using X-ray diffraction, found that the enol of 1,3-diphenyl-1,3-propanedione was slightly asymmetric in accordance with a C_s form.

Determination of the molecular symmetry of the enol forms of β-dicarbonyl compounds has proved rather difficult using spectroscopic methods. However, Musso and Junge[18] have concluded from an elegant ir study using selective ^{13}C substitution that acac does not have C_{2v} symmetry. This result is in direct conflict with the electron diffraction studies. It is clear from NMR investigations[5,19] that, if the enols have C_s symmetry, there is a rapid ($k \geq 10^6$ sec^{-1}) topomerization,[20] so that the molecule appears as C_{2v} symmetric on the NMR time scale.

The present calculation indicates that MA has C_s symmetry. Even if the absolute numbers calculated cannot be quantitatively relied on because of the above-mentioned sources of errors, we feel that the result is conclusive in predicting the occurrence of a double-well potential. It is not clear, of course, to what extent this result can be generalized to other β-dicarbonyl compounds. In acac, the H_2 and H_3 hydrogens of MA have been replaced by methyl groups which are slightly more electron donating. This affects the hydrogen bond but presumably to a minor extent only. A reasonably accurate ab initio calculation on acac would of course be of great value, but the size of this system does not at present make it an attractive candidate for a geometry optimization.

In a semiempirical CNDO/2 calculation on MA, Schuster[21] found an energy difference between the C_s and C_{2v} forms of 2 kJ/mol. We have extended his CNDO/2 calculations to allow for geometrical optimization and obtained qualitatively the same results but with an energy difference of 7 kJ/mol. Gordon and Koob[22] found that the C_{2v} symmetric form was the most stable in an extensive study of acac using the INDO method.

The calculated bond lengths in the C_{2v} form I are in good

Table I. Calculated and Observed Oxygen–Oxygen Distances in Some β-Dicarbonyl Compounds

Compd	Method	O···O distance, Å	Molecular symmetry	Ref
MA	Ab initio	2.63	C_s	This work
MA	Ab initio	2.29	C_{2v}	This work
MA	CNDO/2	2.38	C_s	This work
MA	CNDO/2	2.32	C_{2v}	This work
Acac	INDO	2.28	C_{2v}	22
Acac	Electron diffraction	2.38	C_{2v}	14
Acac	Electron diffraction	2.52	C_{2v}	15
3,4-Diaceto-2,5-hexanedione	X-ray diffraction	2.43	(C_{2v})	13
1,3-Diphenyl-1,3-propanedione	X-ray diffraction	2.47	C_s	12

Table II. Calculated Quadrupole Coupling Constants for a Deuterium Atom in the Hydrogen Bond at Six Points of the Hydrogen Bond Potential Curve (cf. Figure 3)

Reaction coordinate k	q, kHz	Asymmetry parameter	Deviation from bond direction,[a] deg
1.0 (C_{2v})	56	0.33	90[b]
0.75	66	0.59	23.5
0.50	127	0.25	13.5
0.25	209	0.17	7.6
0.0	311	0.12	4.5
−0.25	434	0.10	3.2

[a] This is the deviation of the direction of the largest principal component of the electric field gradient tensor from the bond direction. The largest component is turned clockwise relative to the hydrogen bond defined as in Figure 1, form Ib. [b] The out-of-plane component is the largest!

Table III. Highest and Lowest Calculated Occupied Orbital Energies for the C_s and C_{2v} Forms

C_s symmetry		C_{2v} symmetry	
Energy, eV	Orbital symmetry	Energy, eV	Orbital symmetry
−9.8	A"	−9.7	B_2
−11.4	A'	−11.4	B_1
−14.4	A"	−14.3	A_2
−14.7	A'	−14.6	A_1
−306.0	A'	−305.8	A_1
−308.9	A'	−309.2	B_1
−309.0	A'	−309.2	A_1
−559.8	A'	−560.1	B_1
−560.7	A'	−560.1	A_1

agreement with the values determined by electron diffraction for acac (see Figure 2). However, the experimental values also agree with the mean values over equivalent bonds calculated for the C_s form I. The experimental $C_2C_1C_3$ angle has a value intermediate between the values calculated for the C_{2v} and C_s forms. This trend also appears in the O···O distance, a parameter which has been taken as a measure of the hydrogen bond strength. Some experimental and calculated O···O distances are given in Table I. In two recent electron-diffraction determinations[12,13] of the enol of acac the hydrogen bond was found to be practically linear. This is in marked disagreement with the molecular orbital calculations, and also with the X-ray investigation on 1,3-diphenyl-1,3-propanedione.[10] The angle $O_1H_4O_2$ in the C_{2v} form was calculated to be 155.6, 152.2, and 150.8° using the CNDO/2, INDO, and ab initio methods, respectively, and the X-ray diffraction value was 152°. The results of MO calculations should be more reliable than electron-diffraction data as regards the position of H_4, the hydrogen atom having a comparatively low electron-scattering power. Also, the interpretation of electron-diffraction data necessarily involves a certain amount of ambiguity, especially in this case where minor details may have been obscured by the simultaneous occurrence of the keto tautomer.

Suggestions for Further Experiments

The quadrupole coupling constant q of a deuterium atom in a hydrogen bond is very sensitive to the strength of the bond. The value of q was calculated at the six points on the potential curve shown in Figure 3 and are given in Table II. It is clear that q depends strongly on the value of the reaction coordinate k. A further remarkable feature is that the largest component of the field gradient tensor is out of plane in the C_{2v} symmetric form. It seems clear from these values that it should be possible to determine the symmetry of a hydrogen bonded enol from a measured value of the quadrupole coupling constant.

The technique of photoelectron spectroscopy makes it possible to measure vertical ionization potentials to a good degree of accuracy, and is often a very useful tool in discriminating between different models. However, as seen in Table III, the calculated highest occupied orbital energies are practically the same in the C_s and C_{2v} forms, so that photoelectron spectroscopy would seem to be of limited use in determining which form actually occurs. On the other hand, a difference of 0.9 eV is found between the oxygen 1s orbital energies for the C_s form. It should be possible to observe such an energy difference with the current high-resolution ESCA technique.

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A Novel Approach to the Theory of Catalytic Activity in Terms of Molecular Orbital Mixing

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Abstract: A novel approach is proposed for the estimation of catalytic activity from the change of a frontier orbital, given in terms of general rules which predict the change of a frontier orbital by perturbation of a point charge ("static orbital mixing"), or by overlapping of orbitals ("dynamic orbital mixing") through orbital mixing. These general rules can predict, without calculation, the change of frontier orbitals of a perturbed system from the nonperturbed molecular orbitals. The general rules derived for "static orbital mixing" and for "dynamic orbital mixing" are applied to the Lewis acid-catalyzed Diels-Alder reaction and the Meerwein-Ponndorf reaction, respectively. In both cases, the changes in frontier orbitals and in chemical reactivities under the influence of catalysts could be correctly predicted from the general rules, indicating that this approach should be very useful for the prediction of catalytic reactions. Finally, the meaning of multicenter interactions in catalytic reactions is discussed in connection with enzymatic reactions.

The theory of chemical reactivity has recently been developed so remarkably that organic chemists are able to use it with ease and considerable confidence. This is so mainly because of the simple concepts of the theory, such as HOMO-LUMO interaction in the frontier electron theory developed by Fukui et al.,[2-5] and conservation of orbital symmetry in the Woodward-Hoffmann rule.[6-8]

In this present article, along with the above-mentioned theories, we wish to present a novel approach for the prediction of catalytic activity in terms of molecular orbitals through orbital mixing.

The usefulness of the idea of orbital mixing has been demonstrated. For example, Zimmerman proposed the concept of MO following to draw correlation diagrams and to follow reactions.[9] Recently, Fujimoto and Hoffmann calculated the coefficients of orbital mixings of ethylene oxide and acrolein by perturbation of the electric field in connection with catalytic reaction.[10] Moreover, Libit and Hoffmann[11] showed that orbital mixing due to overlap integrals plays an important role in the charge distribution of a π electron system with methyl substituent by using the perturbation method[12] developed by Imamura, one of the present authors. A trial[13] to illustrate catalytic activity by orbital mixing between orbitals with the same symmetry was also made in a manner similar to the Woodward-Hoffmann method.[6-8]

Our fundamental idea in the present article is as follows. The change in a chemical reactivity by a catalyst is brought about by the change in a frontier orbital concerned, and the change in the frontier orbital should be the result of orbital mixings between the frontier orbital and the other orbitals of the substrate by perturbation due to a catalyst. This perturbation can be classified into two parts, one an electrostatic interaction and the other an overlap of orbitals between the substrate and the catalyst. The former may be called "static orbital mixing" and the latter "dynamic orbital mixing" (see Figure 1). We derive here general rules to predict the static and dynamic orbital mixings. For examples of application, we have chosen the Lewis acid-catalyzed Diels-Alder reaction and the Meerwein-Ponndorf reaction.

Nowadays, molecular orbital concepts have become working tools for most organic chemists. Our qualitative approach may be of practical value not only for the understanding of the essential feature of a catalysis but also for the prediction or design of a catalytic organic reaction.

Formulas for Static Orbital Mixing

Consider a point charge (Ze) interacting with the molecular orbitals of a substrate. The electric field by the point charge can be regarded as a perturbation to the substrate. That is, the perturbed Hamiltonian H' is represented by eq 1

$$H' = -Ze^2/r_{tk} \tag{1}$$

where r_{tk} is the distance between a point charge at t and an electron k. The usual perturbation theory[12,14-17] gives the following equation for the coefficient of orbital mixing d_{ij}

$$\varphi_i(k) = \varphi_i^0(k) + \sum_{j(\neq i)} d_{ij}\varphi_j^0(k) + \ldots \tag{2}$$

$$d_{ij} = \int \varphi_i^0(k) H' \varphi_j^0(k)\, d\tau_k/(\epsilon_i^0 - \epsilon_j^0) = \int \varphi_i^0(k)(-Ze^2/r_{tk})\varphi_j^0(k)\, d\tau_k/(\epsilon_i^0 - \epsilon_j^0) \tag{3}$$

where ϵ_i^0 and ϵ_j^0 are the orbital energies of the ith and jth levels, respectively. The molecular orbital, $\varphi_i^0(k)$, may generally be represented by the linear combination of atomic